B.C.A Question Bank 2024-25

Mathematical Foundation

For 2 Marks

Unit-1

(1) Define Minors, Cofactors

(2) Properties of Determinants.

(3) Find the value of $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

(4) What is Matrix?

(5) If $A = \begin{bmatrix} 1 & 0 & 5 \\ 3 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix}$ find A+B.

(6) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$ find 2A-3B

Unit-2

(1) Write the statement of Cayley-Hamilton Theorem.

(2) Define Eigen value and eigen vector.

(3) Find eigen values of the matrix, $A = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$

Unit-3

(1) Differentiate $\dfrac{5+4\cos x}{\sin x}$.

(2) Differentiate $\sqrt{\tan 3x}$

(3) Write the Leibnitz's Theorem.

(4) If $u = ax^2+by^2$ and $v = (ax^2-by^2)$, then find the value of $J\left(\dfrac{u,v}{x,y}\right)$.

Unit-4

(1) Define Gamma function.

(2) Define Beta functions

(3) write the relation between Beta and Gamma function.

(4) write the formula of Integration by Parts.

(5) Evaluate $\int x \sin x \, dx$

(6) Evaluate $\sqrt{\frac{7}{2}}$.

Unit-5

(1) Define Laplace Transform.

(2) write the statement of Existence theorem.

(3) what is first shifting theorem and second shifting theorem.

(4) Define Connectivity.

(5) Define Graphs.

For 7 Marks

Unit-1

(1) Prove that $\begin{vmatrix} a+b+c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

(2) Find Inverse of matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 5 & -7 \\ 3 & 2 & -1 \end{bmatrix}$.

(3) Find the rank of the matrix

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$$

(4) Find the rank of matrix using Echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 5 \end{bmatrix}$$

5. Solve by using Gauss-Jordan elimination method $x+2y+3z=1$, $2x+4y+5z=2$, $3x+5y+6z=4$

6. Find the inverse of the matrix using elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

7. If $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & -3 & -1 \\ 1 & 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 5 \\ 1 & 1 & 0 \end{bmatrix}$

then find the value of $3A-B+5C$.

Unit-2

1. Show that the system of equation $x+y+z=-3$, $3x+y-2z=-2$, $2x+4y+7z=7$ is not consistent.

2. Find the eigen values and eigen vectors of the following matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$.

3. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}.

4. Prove that the vectors $(2,3,4)$, $(0,1,2)$, $(-1,1,-1)$ linearly independent.

5. Test the linear dependency and find the relationship if it exist for $x_1 = (1,1,1,3)$, $x_2 = (1,2,3,4)$, $x_3 = (2,3,4,7)$.

(6) Determine, for what value of λ and μ the following system of equations.
$x+y+z=6, \quad x+2y+3=10, \quad x+2y+\lambda z=\mu$
have (i) no solution (ii) unique solution
(iii) Infinite solution

(7) Find Nullity of matrix $A=\begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix}$

Unit-~~4~~

(1) Prove that $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

(2) Prove that $B(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx$

(3) Prove that $\overline{|n+1} = n!$

(4) Prove that $\overline{|n+1} = n\overline{|n}$

(5) Prove that $B(m,n) = B(n,m)$

(6) Prove that Relation between Beta and Gamma function $B(m,n) = \frac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$.

(7) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$

(8) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$

(9) Solve $(D^2+5D-6)y = \sin 3x + \cos 2x$

(10) Evaluate $\int x^2 \sin x \, dx$

Unit-3

(1) If $y = a\cos(\log x) + b\sin(\log x)$

Prove that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2+1)y_n = 0$

(2) If $y = e^{m\cos^{-1}x}$, Show that

$(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (n^2+m^2)y_n = 0$

Calculate $y_n(0)$

(3) If $u = \log(x^3+y^3+z^3-3xyz)$, Show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(4) If $u = \dfrac{x^3+y^3}{x+y}$, then find the value of

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

(5) If $z = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, then Prove that

$x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = \tan z$.

(6) If $u = \log\left(\dfrac{x^5+y^5+z^5}{x^2+y^2+z^2}\right)$, Prove that

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3$

(7) If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ and $\dfrac{\partial(r,\theta)}{\partial(x,y)}$

Also Prove that $JJ' = 1$

(8) If $u = x^2 - y^2$, $v = 2xy$ find $\dfrac{\partial(x,y)}{\partial(u,v)}$.

(9) If $u = x^y$, then show that $\dfrac{\partial^3 u}{\partial x^2 \partial y} = \dfrac{\partial^3 u}{\partial x \partial y \partial x}$.

Unit-5

(1) Find the Laplace transform of

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ t, & 1 \leq t < 2 \\ t^2, & 2 \leq t < \infty \end{cases}$$

(2) Find the laplace transform of $L = \{\cosh^3 2t\}$.

(3) Find the Laplace transform of the function $L\{t \sin^2 3t\}$.

(4) Find the Laplace transform of $e^{-3t}(\cos 4t + 3\sin 4t)$.

(5) Find $L\{F(t)\}$ if $F(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$

(6) Find the Laplace transform of $\int_0^t e^{-t} \cos t \, dt$

(7) Express the functions in terms of unit step function and find its laplace transform.

$$f(t) = \begin{cases} 0, & 0 < t < \frac{\pi}{2} \\ \sin t, & t > \frac{\pi}{2} \end{cases}$$

(8) Explain the following

(I) Connectivity (II) Adjacency matrix

(III) Hamilton graphs.